

## From Total Energies to Curie Temperatures (with ABINIT and VASP)

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Ferromagnetic semiconductors  
From DFT to Heisenberg model  
Percolation theory for  $T_c$   
ABINIT vs. VASP  
Self-compensation and  $T_c$





# What are ferromagnetic semiconductors?

## Prehistory

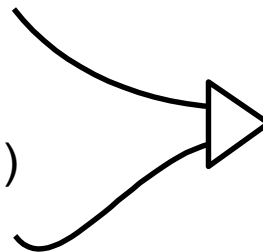
- ◆ 1960s: Crystalline FM semiconductors; transition-metal chalcogenides, spinels

## Ancient history

- ◆ 1980s: Dilute Magnetic Semiconductors based on II-VI hosts; Mn-doped CdTe

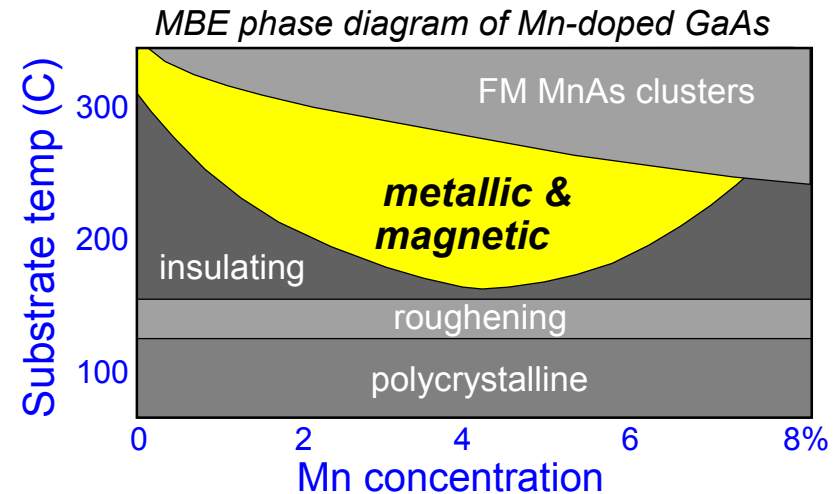
## Ferromagnetic DMS's

- ◆ 1989: Mn-doped InAs (IBM)
- ◆ 1996: Mn-doped GaAs (Tohoku)
- ◆ 2000: Mn-doped Ge (NRL)



**All new!**

*Non-volatile character  
plus  
Wavefunction engineering*



## Caveats

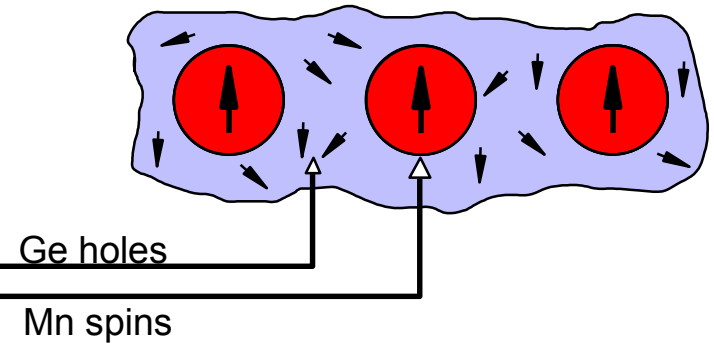
- Curie temperatures limited to 120 K
- Limited to *p*-type conduction; *n*-type needed



# Ferromagnetism from indirect exchange

## *The Zener Model for ferromagnetic semiconductors*

$$H = H_0 + J_{pd} \sum \vec{S}_I \cdot \vec{s}_i$$

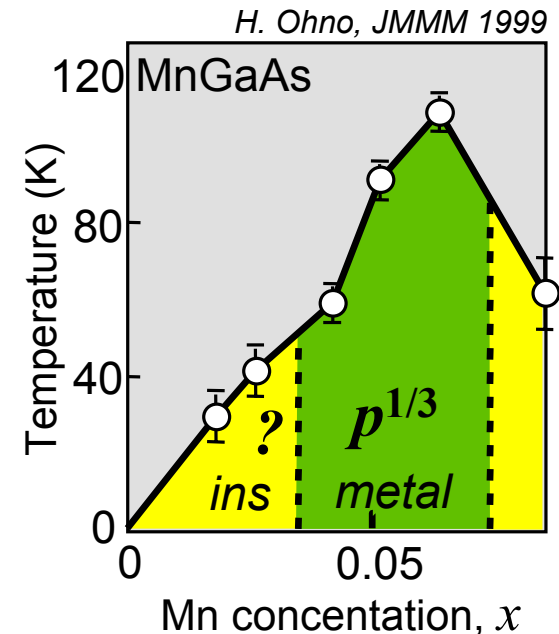


### *Important points*

- ◆ Ge(p)-Mn(d) hybridization  $\Rightarrow J_{pd} > 0$  (Antiferromagnetic)
- ◆ Effective interaction between Mn spins is **Ferromagnetic**
- ◆ Mn provide both spins,  $x$ , and holes,  $p$

Mean-field Curie temperature for  $\text{Mn}_x\text{Ge}_{1-x}$

$$T_c \propto J_{pd}^2 x p^{1/3} S(S+1)$$

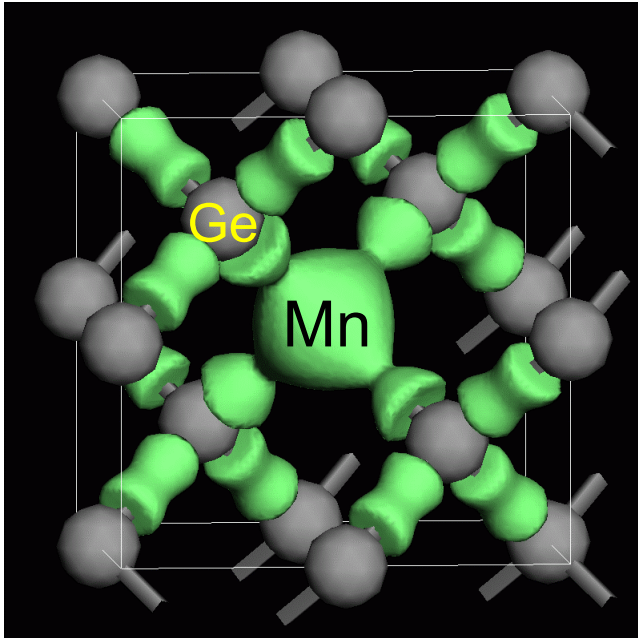




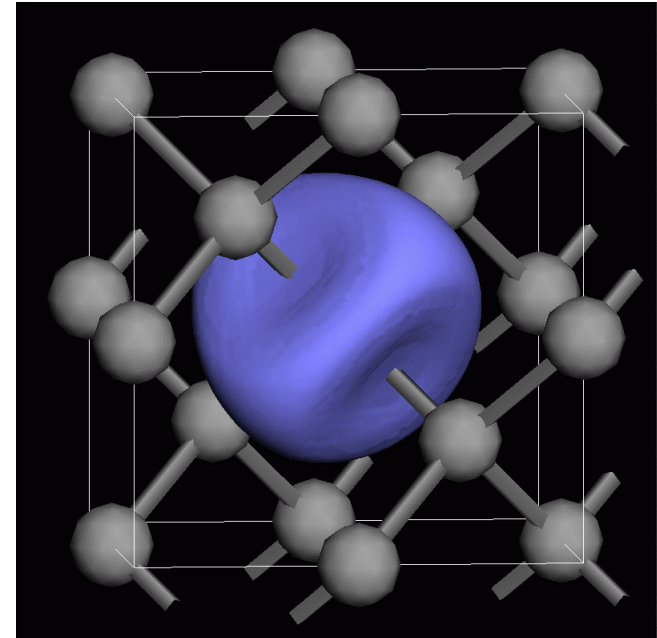
# Elemental FM semiconductor, $\text{Mn}_x\text{Ge}_{1-x}$

Y.D. Park, A. Hanbicki, S.C. Erwin, C.S. Hellberg, J.M. Sullivan, ..., B.T. Jonker, *Science* **295**, 651 (2002)

*Valence electron density*



*Electron spin density*



Density  
Functional  
Theory

- ◆ Mn strongly prefers the substitutional site
- ◆ Surrounding Ge lattice nearly undistorted (< 2% bond strain)
- ◆ Spin density completely localized around Mn



# From DFT to the Heisenberg model

## Two Mn atoms in Ge supercell

- ◆  $4 \times 2 \times 2 \Rightarrow 32$  atoms, i.e. 6% Mn

## DFT total energies

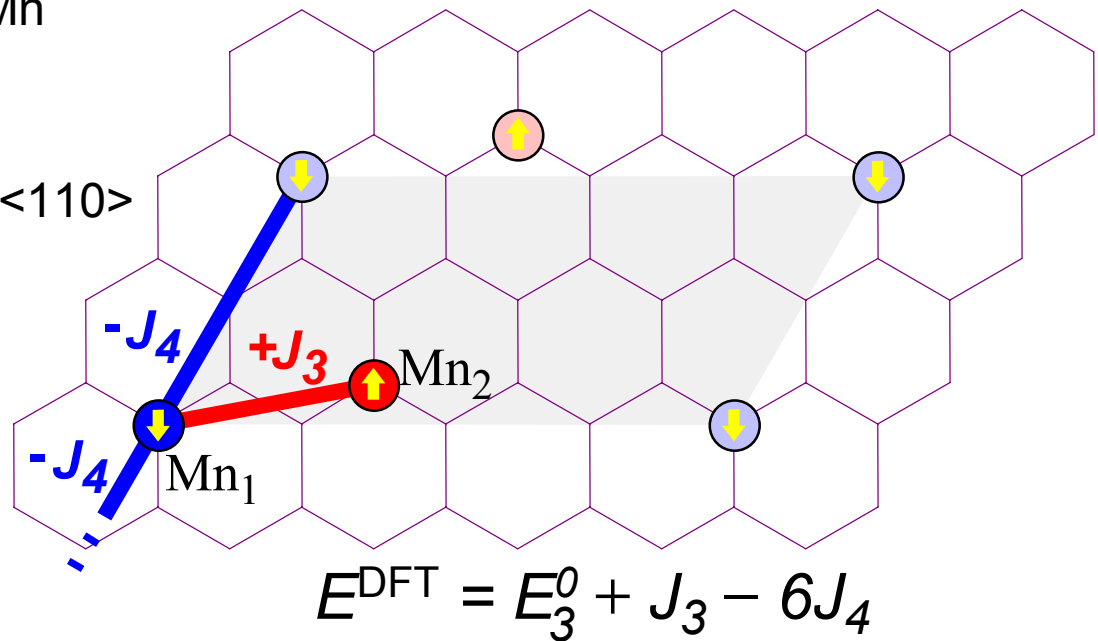
- ◆ 4 arrangements of Mn along  $\langle 110 \rangle$
- ◆ 2 spin configurations

## Heisenberg Hamiltonian

$$H = E_{ij}^0 - \sum J_{ij} \vec{S}_i \cdot \vec{S}_j$$

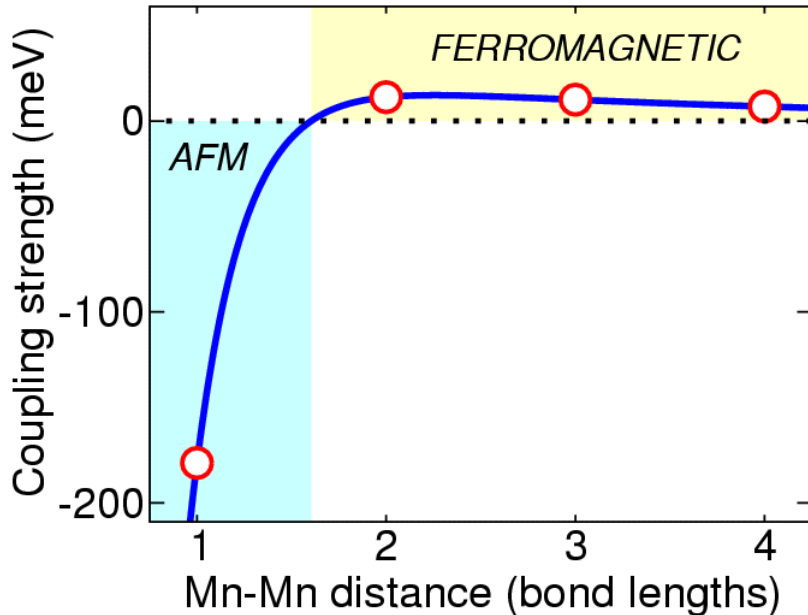
## Coupled linear equations

- ◆  $\Rightarrow$  Spin coupling constants,  $J_n$
- ◆  $\Rightarrow$  Chemical energies,  $E_n^0$





# Spin coupling constants



## *DFT coupling constants*

$J_1$	-179.1 meV
$J_2$	+12.4
$J_3$	+11.1
$J_4$	+7.6

## *Four-parameter fit*

$$J(r) = J_{AF} \exp(-r/R_{AF}) + J_{FM} \exp(-r/R_{FM})$$

## *Competing interactions*

$$\Rightarrow R_{AF} = 0.5 \text{ \AA}, R_{FM} = 5.0 \text{ \AA}$$

*Strong short-range AFM competing with weak long-range FM*



# Ferromagnetic phase transition

## Mean-field theory assumptions

- ◆ Each spin interacts with many
- ◆ Interaction range  $\gg$  spin radius

## MnGe exp'tal concentrations

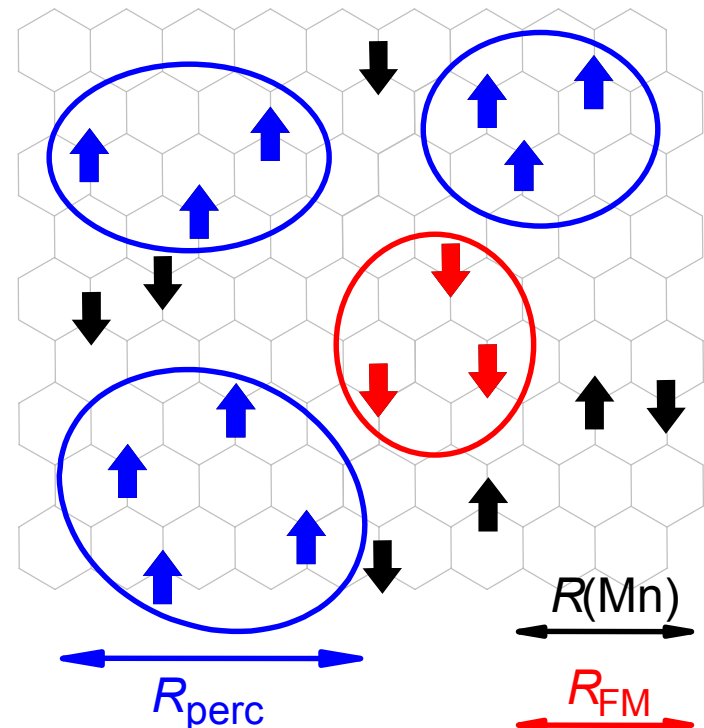
- ◆ Ferromagnetic range  $R_{\text{FM}} = 5.0 \text{ \AA}$
- ◆ Average Mn separation comparable:  
 $R(\text{Mn}) \sim 1/x^{1/3}$  (4.6  $\text{\AA}$  for  $x=0.05$ )

## Percolation theory for $T_c$

Ferromagnetic transition occurs when FM clusters reach  $R_{\text{perc}} = 1.34 R(\text{Mn})$ .

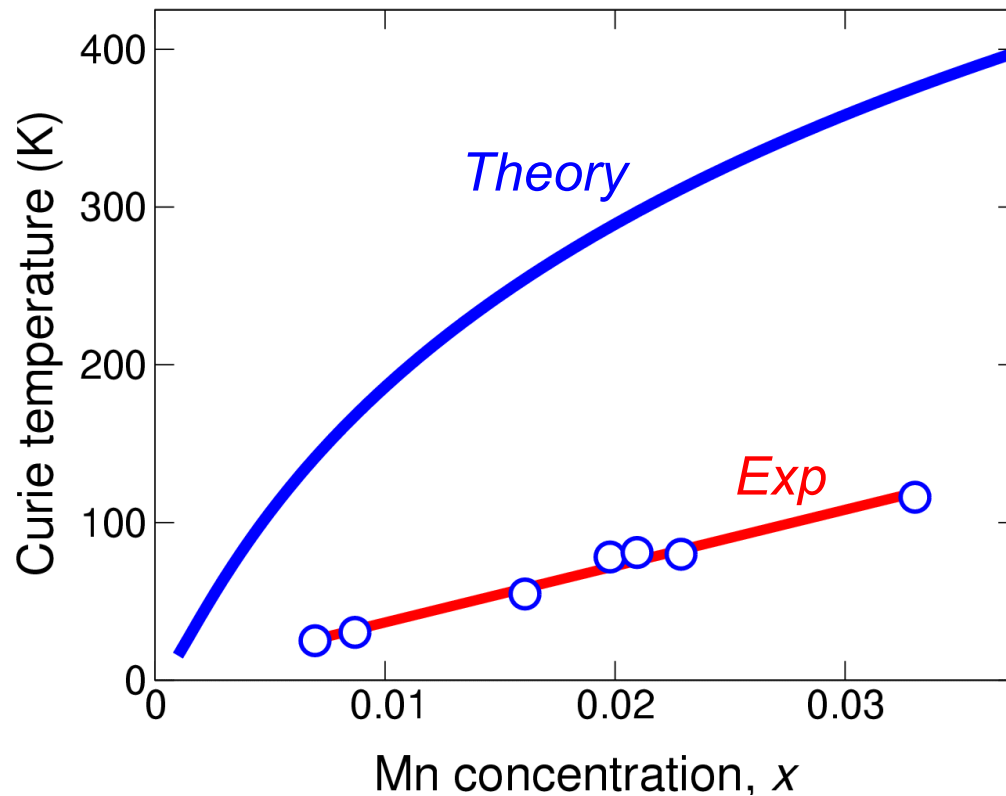
Coupling strength then determines  $T_c$  according to:

$$k_B T_c = S(S+1) J(R_{\text{perc}})$$





# Curie temperature vs. Mn concentration

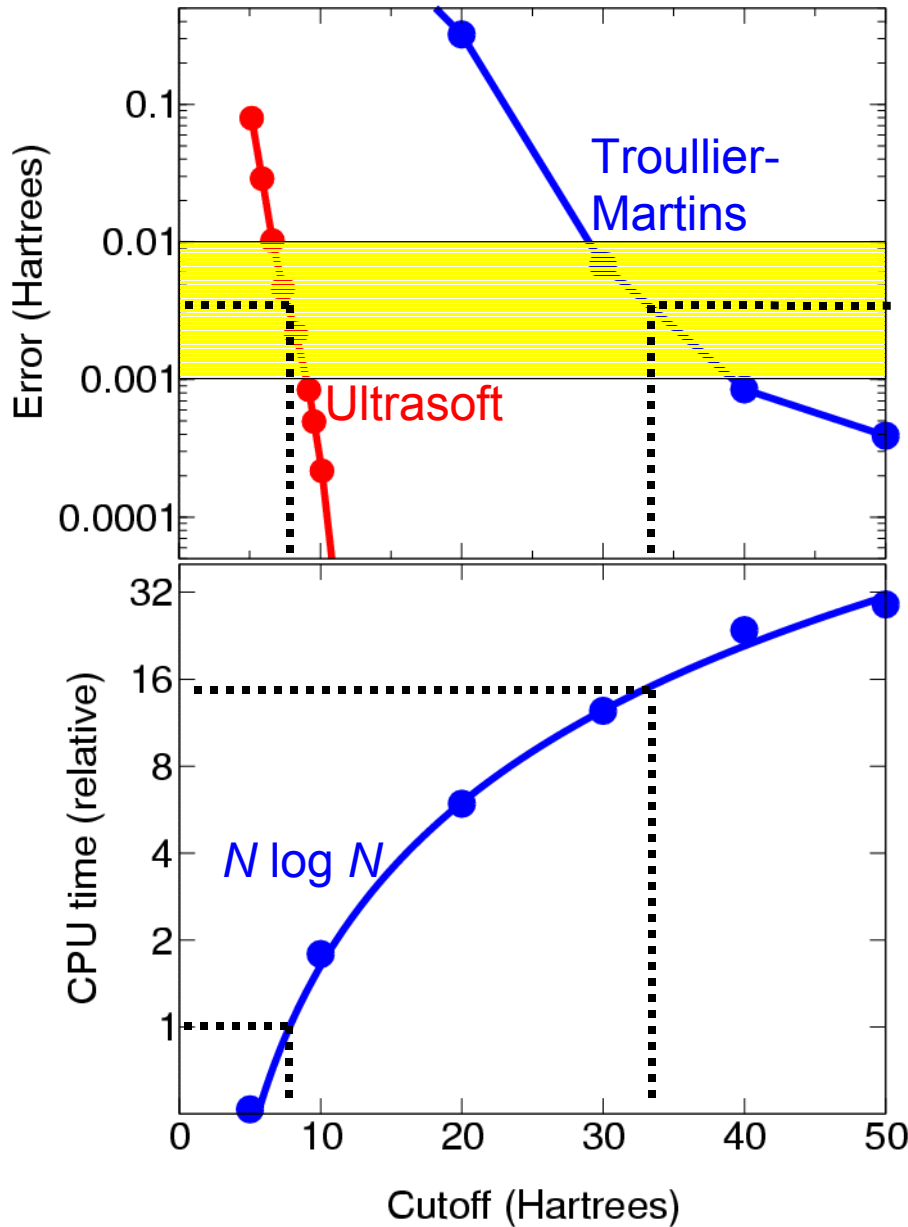


- 1. Known tendency of LDA to overestimate  $J$  (by 3-6)
- 2. Strong crystallographic anisotropy of  $J(\vec{R})$
- 3. Hole compensation (in the material, but not in DFT)





# ABINIT vs. VASP



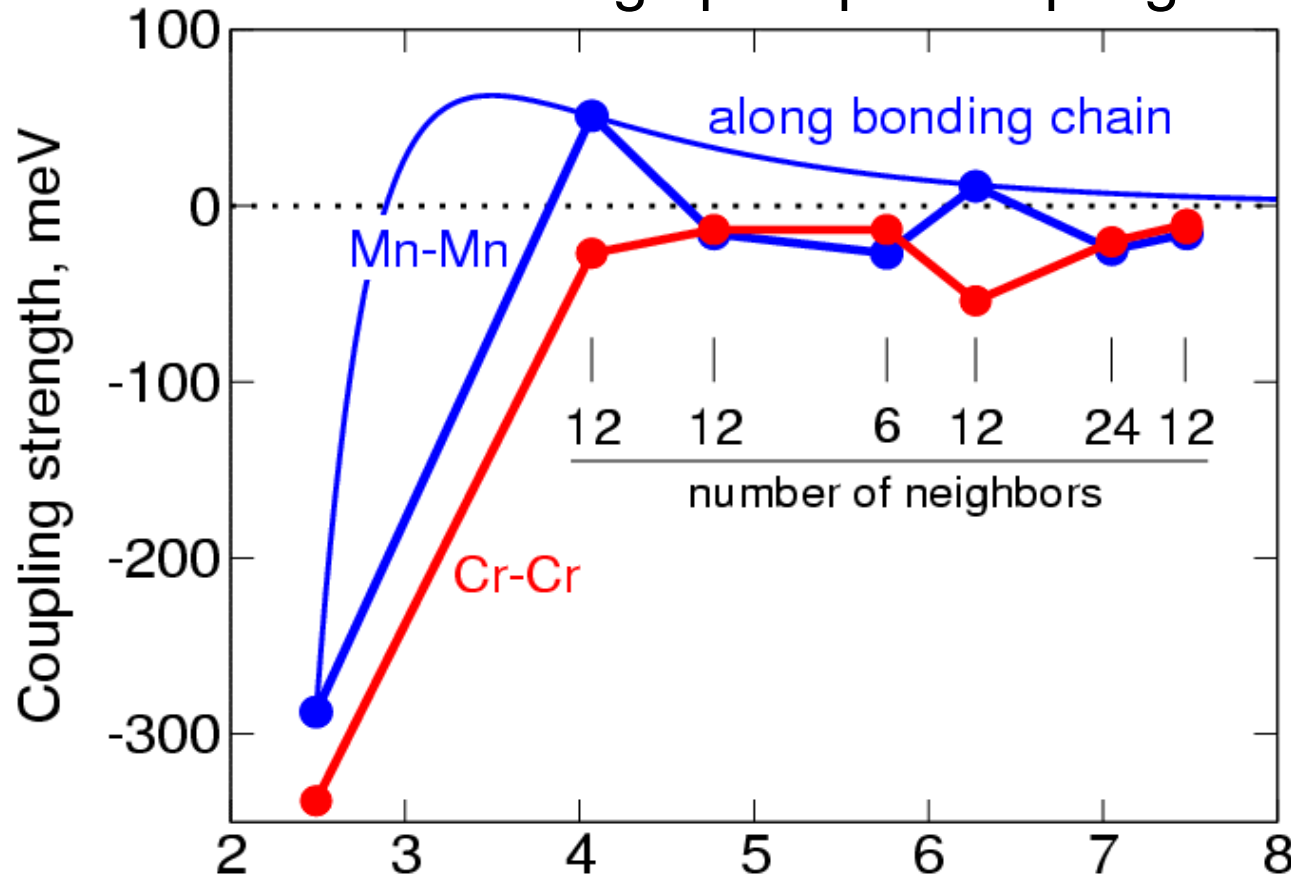
## *Soft vs. ultrasoft psp's*

- Test: Total energy of  $\text{Mn}_1\text{Ge}_{15}$
- ABINIT: Troullier-Martins (\*.pspnc)
- VASP: Ultrasoft (PAW is similar)
- CPU time dominated by FFT
- Ultrasoft advantage > factor 10 (!)



# Spin coupling constants, revisited

## Heisenberg spin-spin coupling



- 128-atom supercells
- DFT total energies
- Pseudopotentials

MnGe

FM beyond n.n.

FM and AF up to 8 Å

CrGe

AF everywhere  $\Leftrightarrow$  exp

*Large crystallographic anisotropy can make FM-vs-AF competition more important than previously suspected*

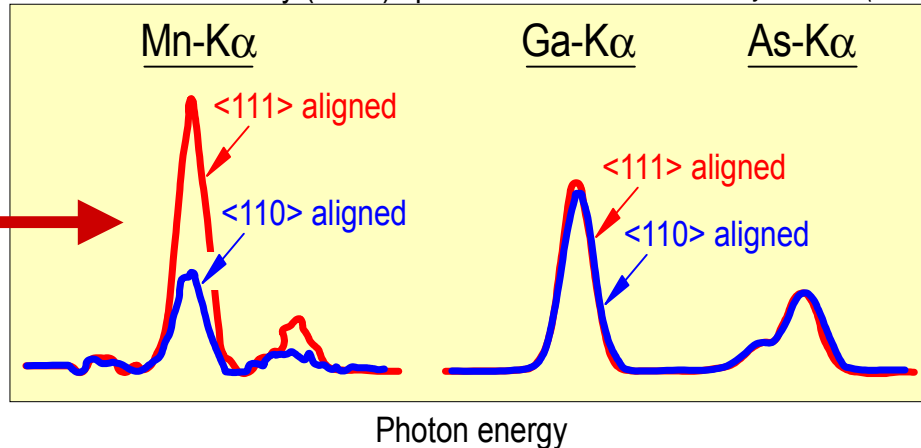


# Compensation by Mn interstitials

S.C. Erwin & A.G. Petukhov, PRL (2002)

Particle-induced x-ray (PIXE) spectra

Yu et al., Phys. Rev. B (2002)



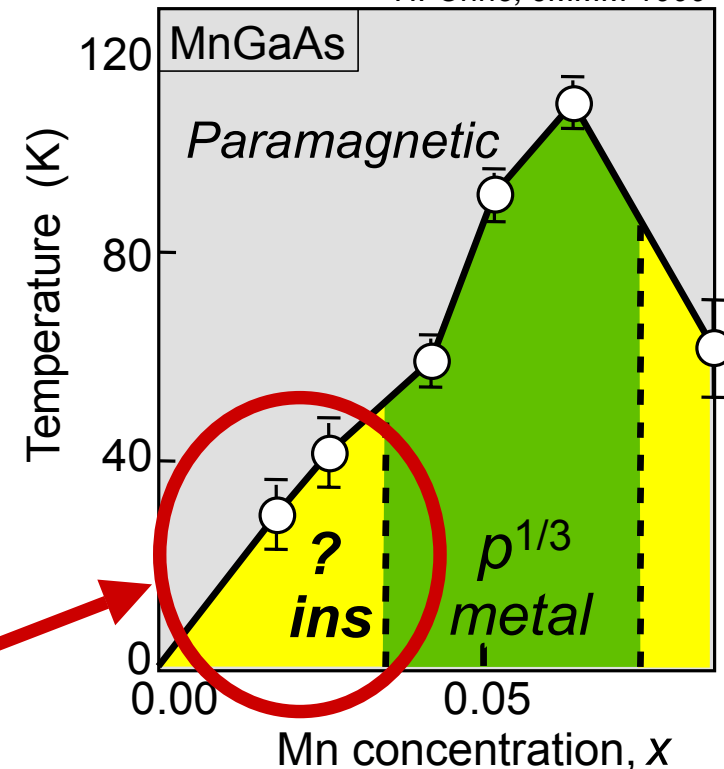
## Questions for theory

1. How/why are interstitials formed?
2. What determines relative abundance of substitutional and interstitial Mn?
3. Under what conditions do interstitials compensate?
4. What role does compensation play in the ferromagnetism?

## Experimental findings

- Compensation: MnGaAs, MnGe
- Ion channeling  $\Rightarrow$  Interstitial Mn!
- Exp'l correlation:  $T_c \Leftrightarrow p \Leftrightarrow x_{\text{int}}$

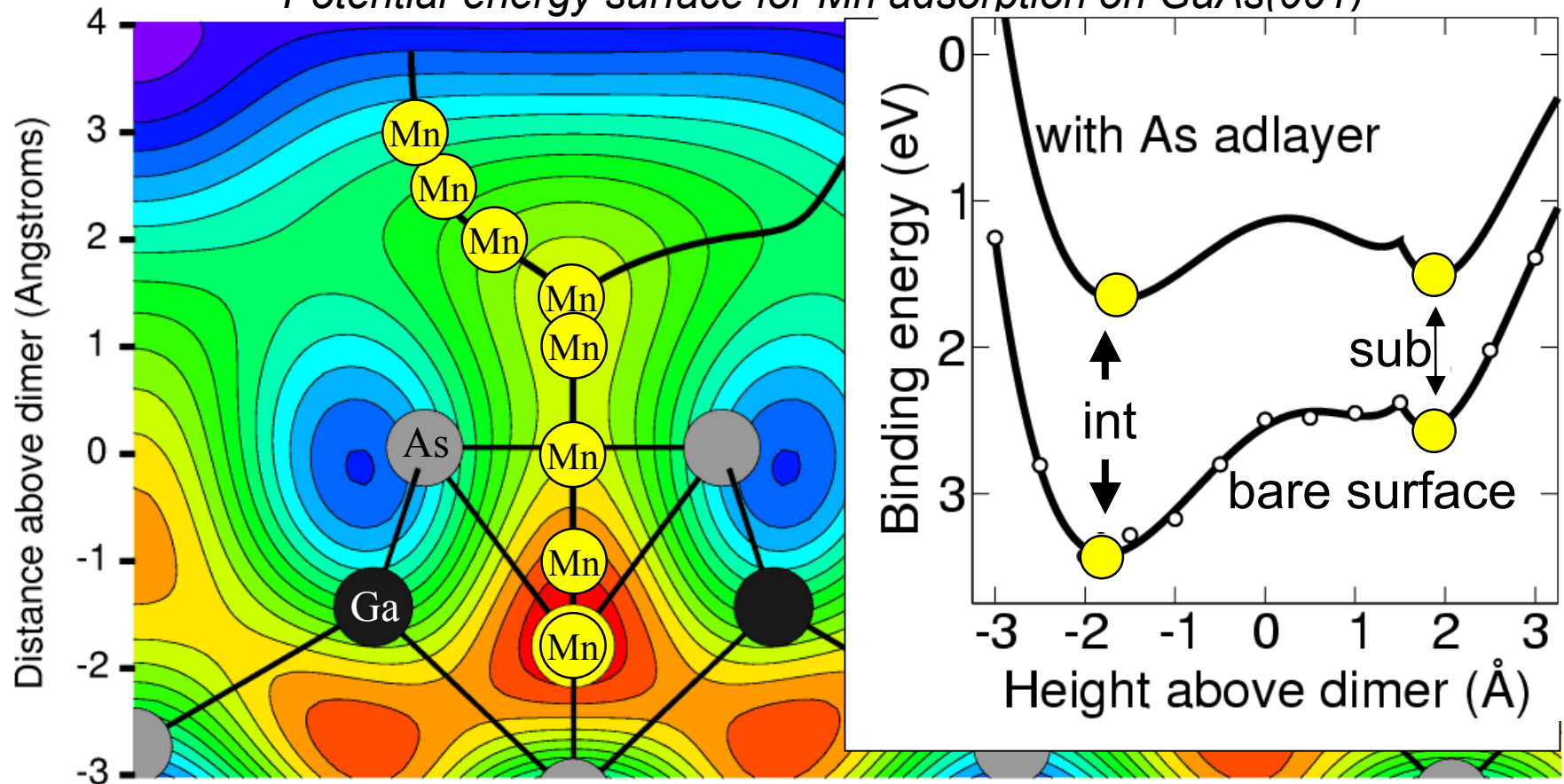
H. Ohno, JMMM 1999





# Interstitials form during MBE growth

*Potential-energy surface for Mn adsorption on GaAs(001)*



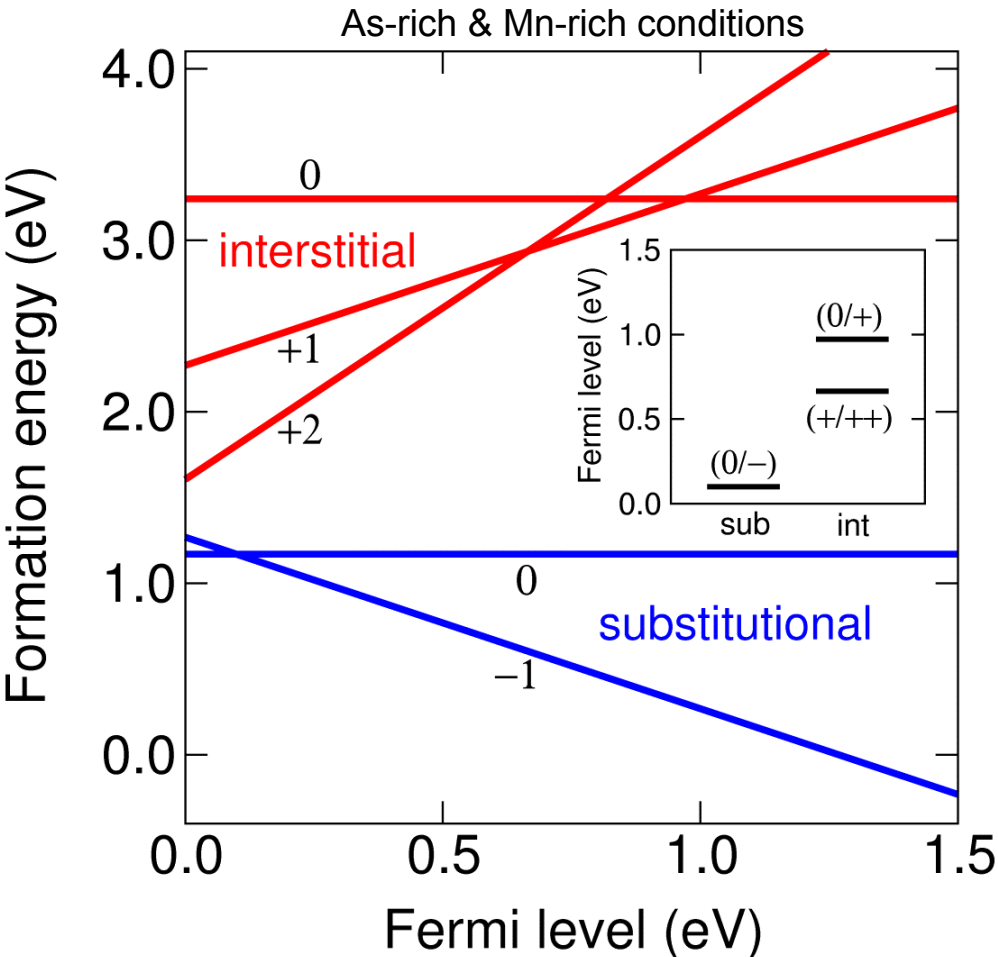
Adsorption at low temperature leads to substitutional and interstitial Mn



# Electrical activity of Mn impurities

Formation energy

$$E_f[\text{Mn}^q] - n_{\text{Ga}}\mu_{\text{Ga}} - n_{\text{As}}\mu_{\text{As}} - \mu_{\text{Mn}} + qE_F$$



◆ Substitutional Mn is a *single acceptor*

- Ionization energy = 100 meV
- Exp. ionization energy = 113 meV

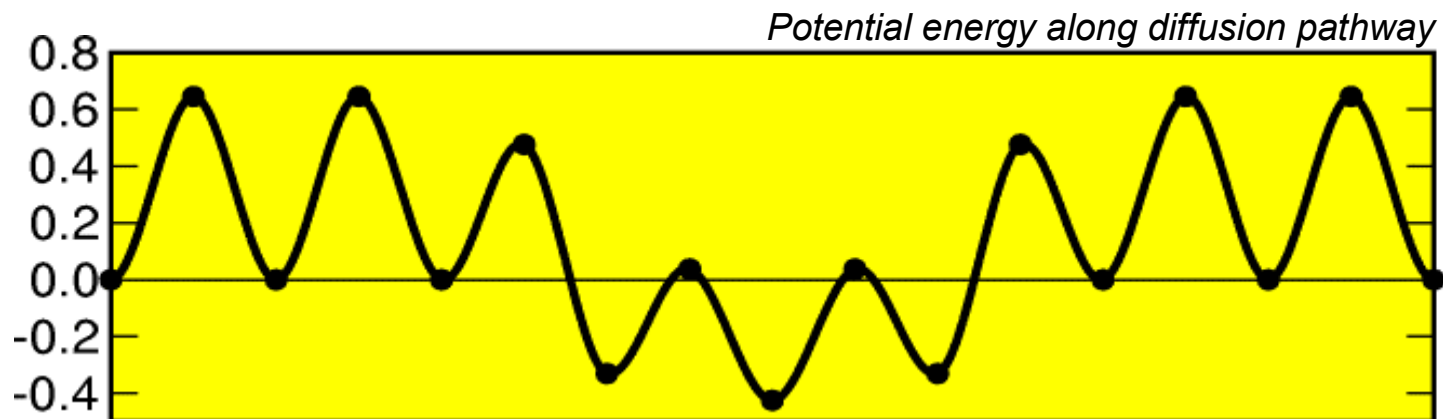
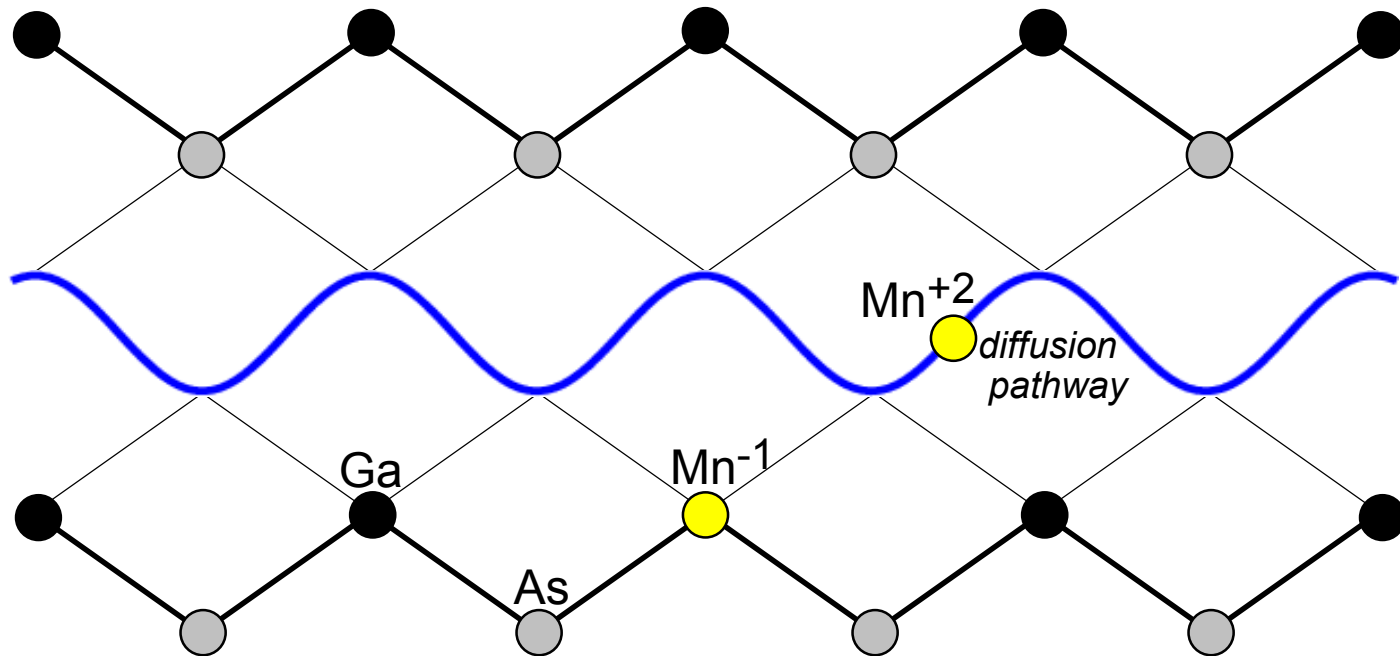
◆ Interstitial Mn is a *double donor*

Each interstitial compensates  
two substitutional Mn impurities

If 1/3 of the Mn is interstitial,  
material is completely compensated



# What prevents interstitials from escaping?





# Consequences for magnetism

## Zener Model revisited

$$H = \sum t_{ij} a_{i\sigma}^\dagger a_{j\sigma} - J_{pd} \sum \vec{S}_i \cdot \vec{s}_i + \sum \epsilon_i a_{i\sigma}^\dagger a_{i\sigma}$$

Hopping of holes  
between Mn spins

Exchange coupling  
between Mn spin  
and hole

Fluctuations in on-site energies  
due to neutral and charged  
acceptors,  $\text{Mn}^0$  and  $\text{Mn}^{-1}$

2nd-order  
perturbation  
theory

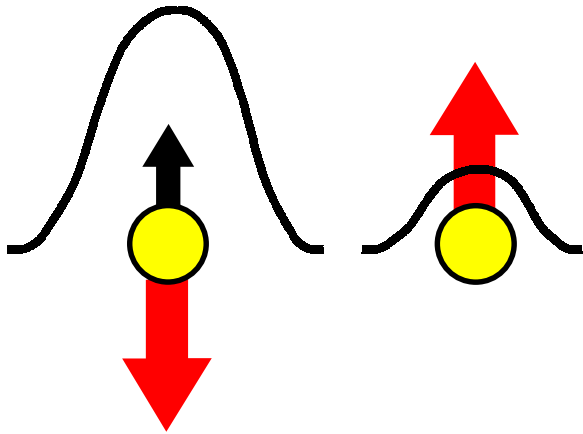
Effective Heisenberg interaction

$$H = \sum \frac{|t_{ij}|^2}{\epsilon_i - \epsilon_j} (q_i - q_j) \left( 1 + \frac{\vec{S}_i \cdot \vec{S}_j}{S(S+1)} \right)$$

$\Rightarrow$  Ferromagnetic interaction survives  
only between  $\text{Mn}^0$  and  $\text{Mn}^{-1}$ !

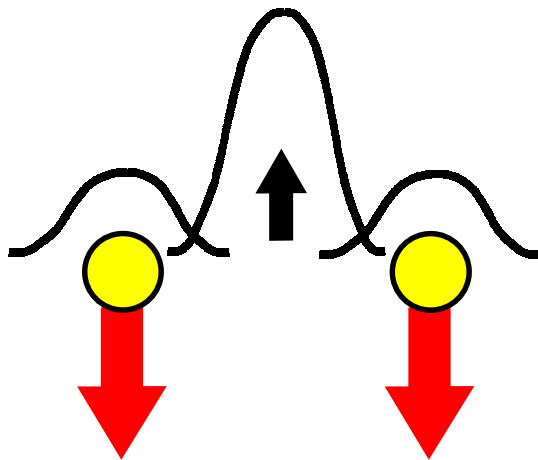


# Toy model: Two Mn atoms sharing one hole



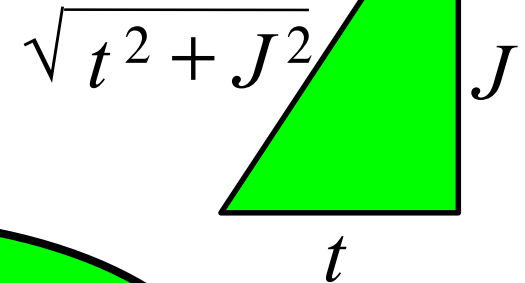
*Antiferromagnetic*

$$E_{\text{AF}} = -\sqrt{t^2 + J^2}$$



*Ferromagnetic*

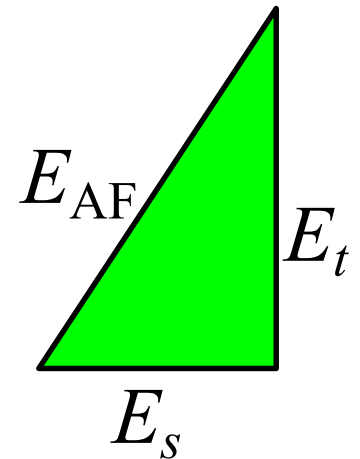
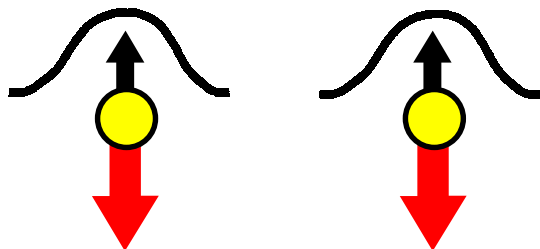
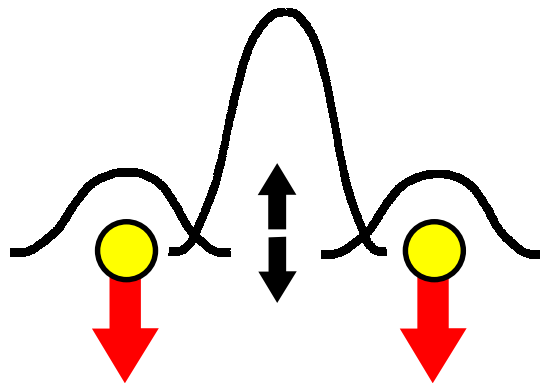
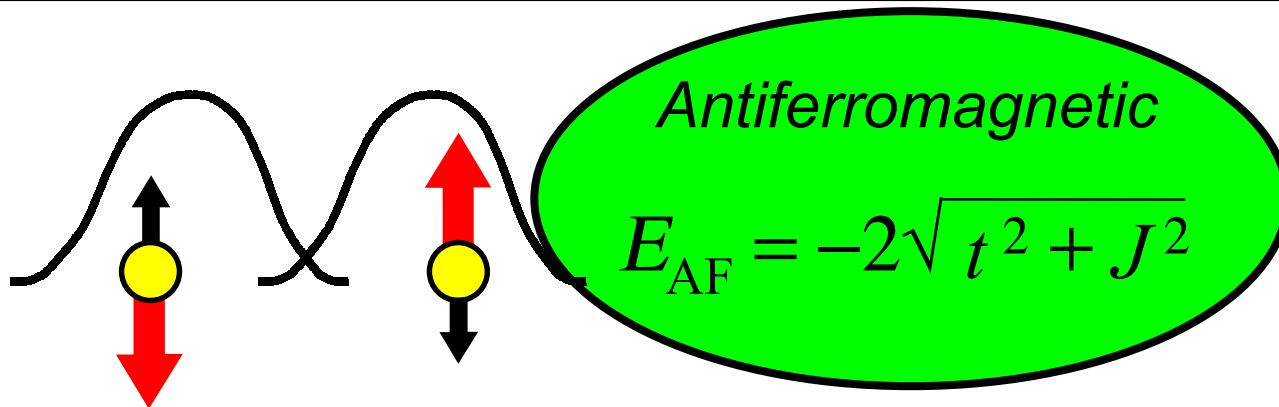
$$E_{\text{FM}} = -|t| - |J|$$







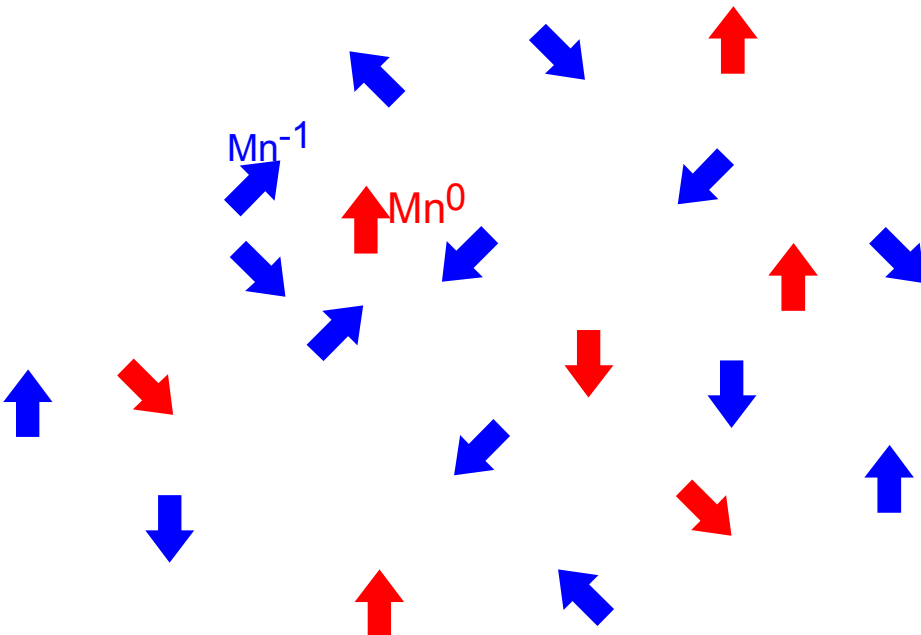
# Toy model: Two Mn atoms sharing two holes





# FM order from two-color percolation

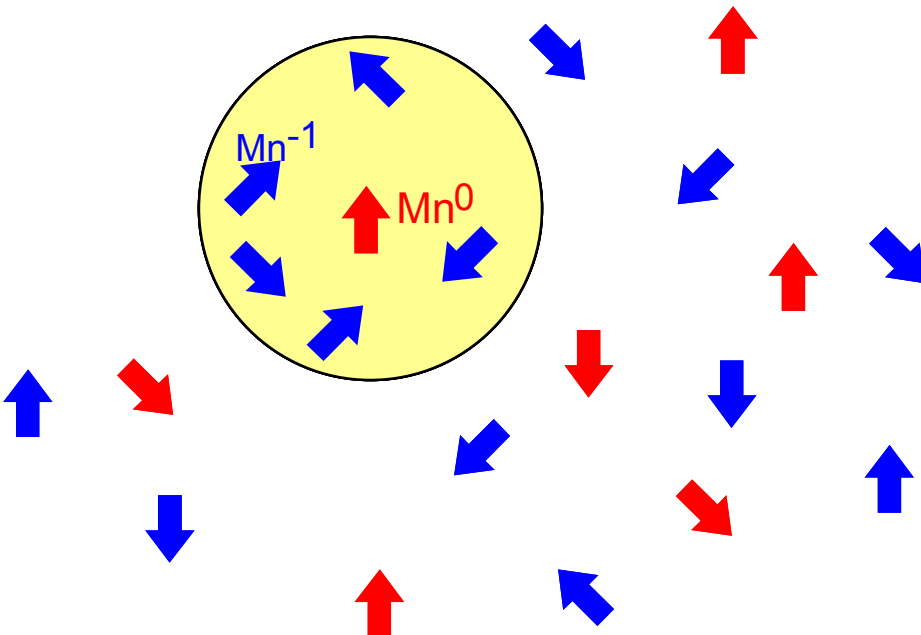
1. Randomly oriented Mn spins, each with charge 0 or -1





# FM order from two-color percolation

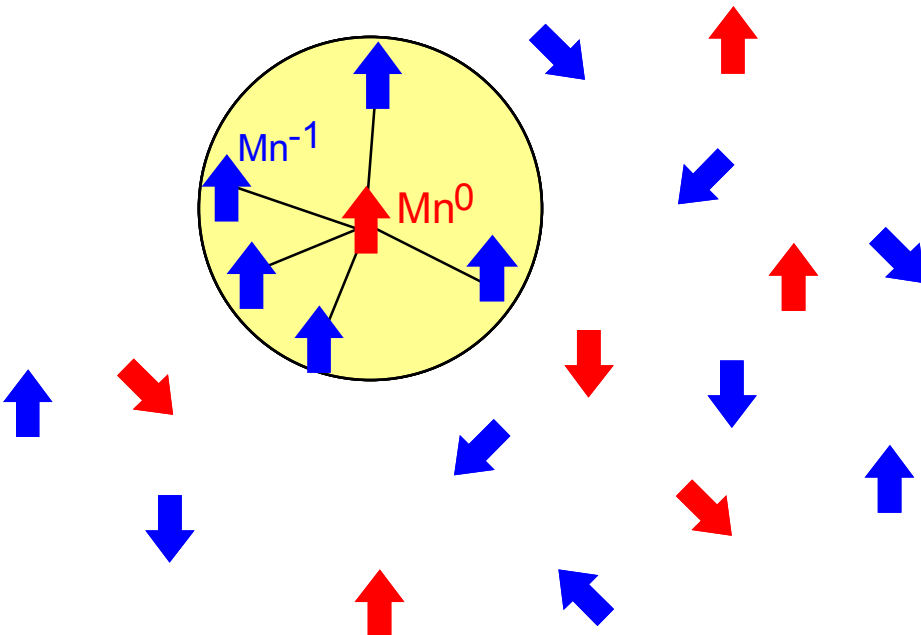
1. Randomly oriented Mn spins, each with charge 0 or -1
2. Spins interact if their wavefunctions overlap





# FM order from two-color percolation

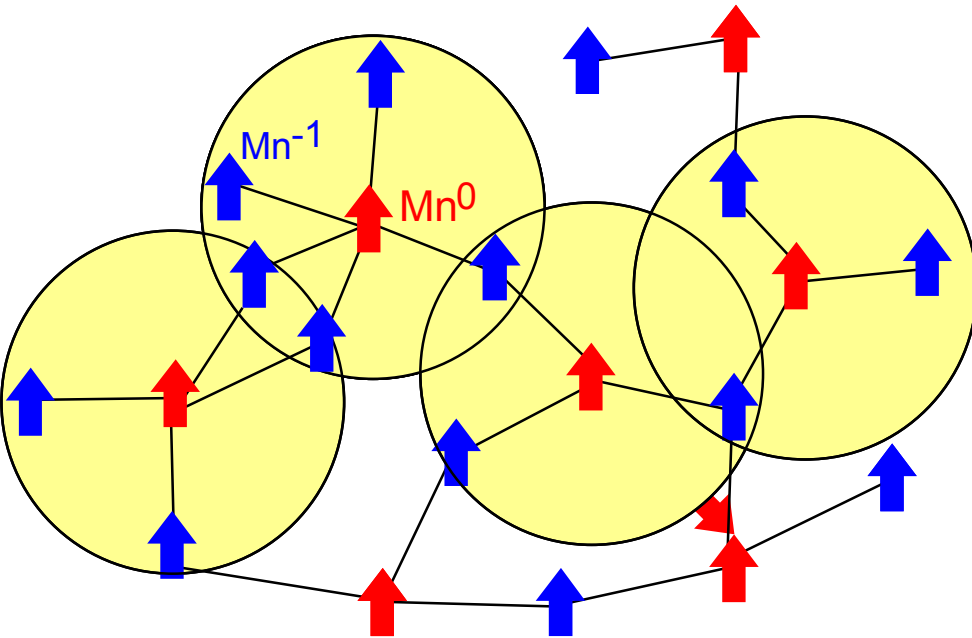
1. Randomly oriented Mn spins, each with charge 0 or -1
2. Spins interact if their wavefunctions overlap
3. Spins with opposite charge interact to form ferromagnetic clusters





# FM order from two-color percolation

1. Randomly oriented Mn spins, each with charge 0 or -1
2. Spins interact if their wavefunctions overlap
3. Spins with opposite charge interact to form ferromagnetic clusters
4. If clusters are larger than the percolation threshold, the entire spin system becomes ferromagnetic



$$R_{\text{perc}} \propto \left( \frac{1}{\sqrt{q(1-q)}} \right)^{1/3}$$

diverges when:

all  $\text{Mn}^0$   
(zero comp)

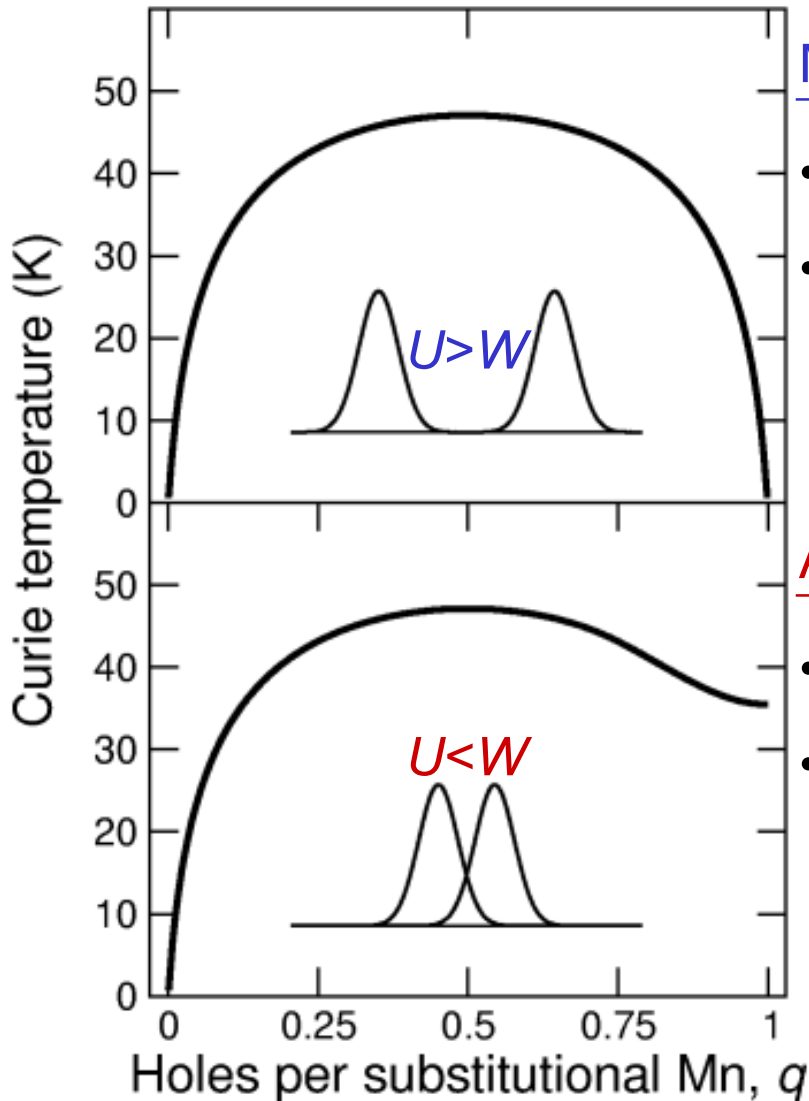
all  $\text{Mn}^{-1}$   
(complete comp)



# Curie temperature vs. compensation

Fraction of interstitial Mn,  $y/x$

1/2      1/4      0



## Mott-Hubbard insulator

- Zero compensation:  $T_c = 0$  !
- Curie temperature highest for  $q=0.5$

## Anderson localization

- Zero compensation:  $T_c$  suppressed !
- Curie temperature highest for  $q=0.5$

*Below the metal-insulator transition, partial compensation in Mn-doped semiconductors promotes ferromagnetism!*



# Lessons Learned

1. Convergence is not the same as completeness.
2. Completeness is impossible, but one must try.
3. DFT can't do everything!

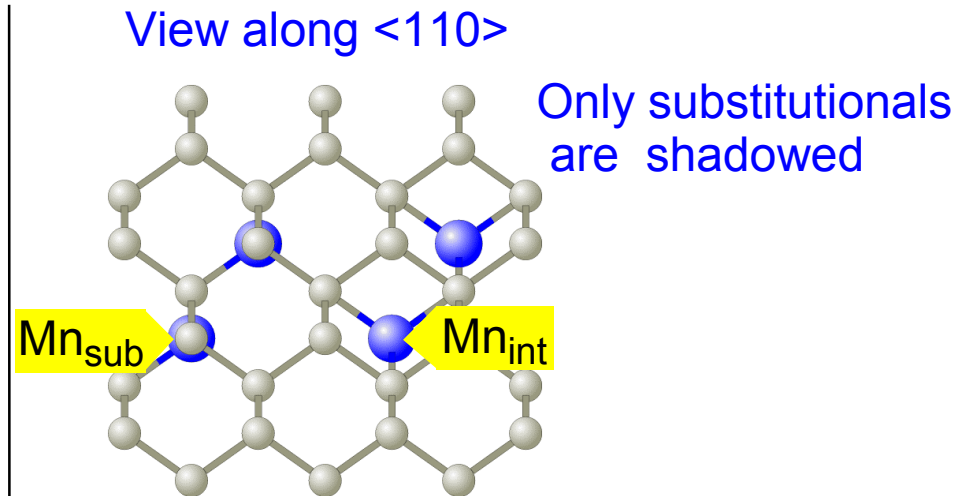
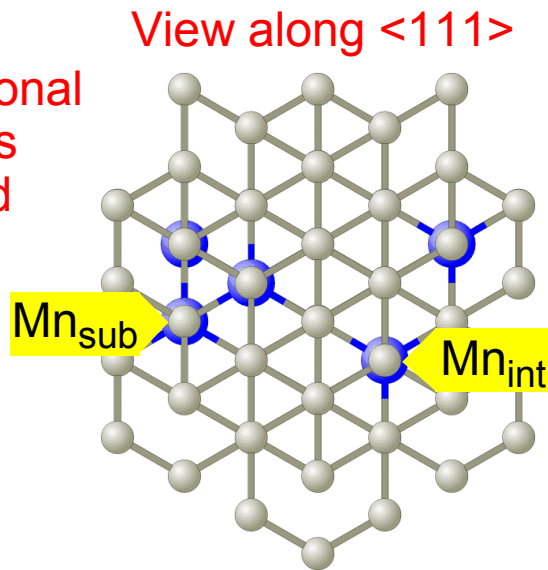






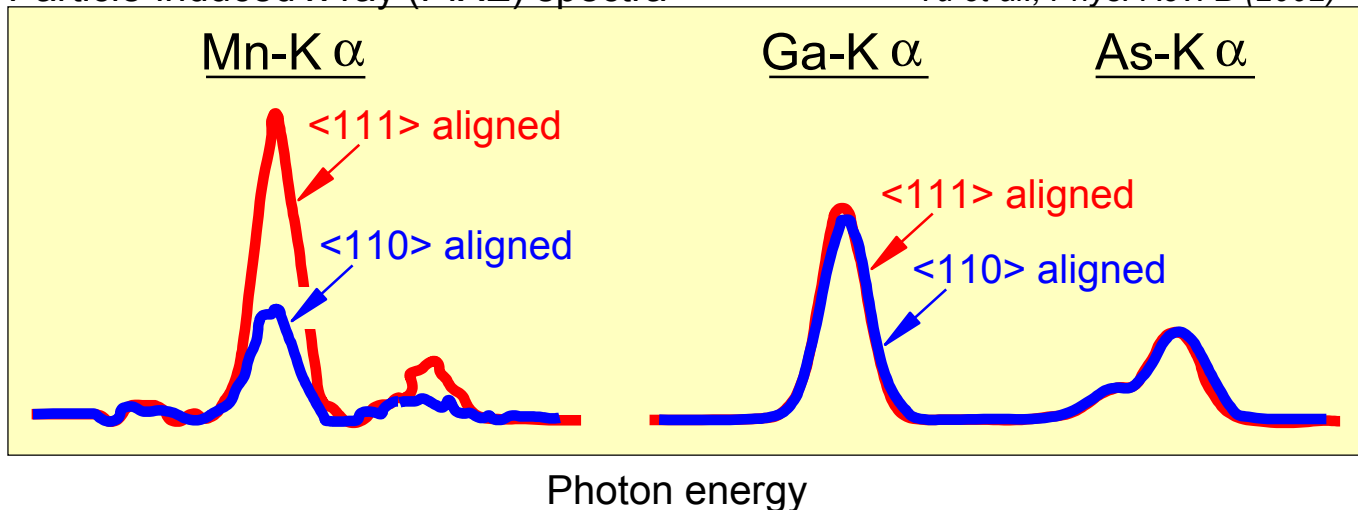
# Experimental evidence for interstitial Mn

Both substitutional and interstitials are shadowed



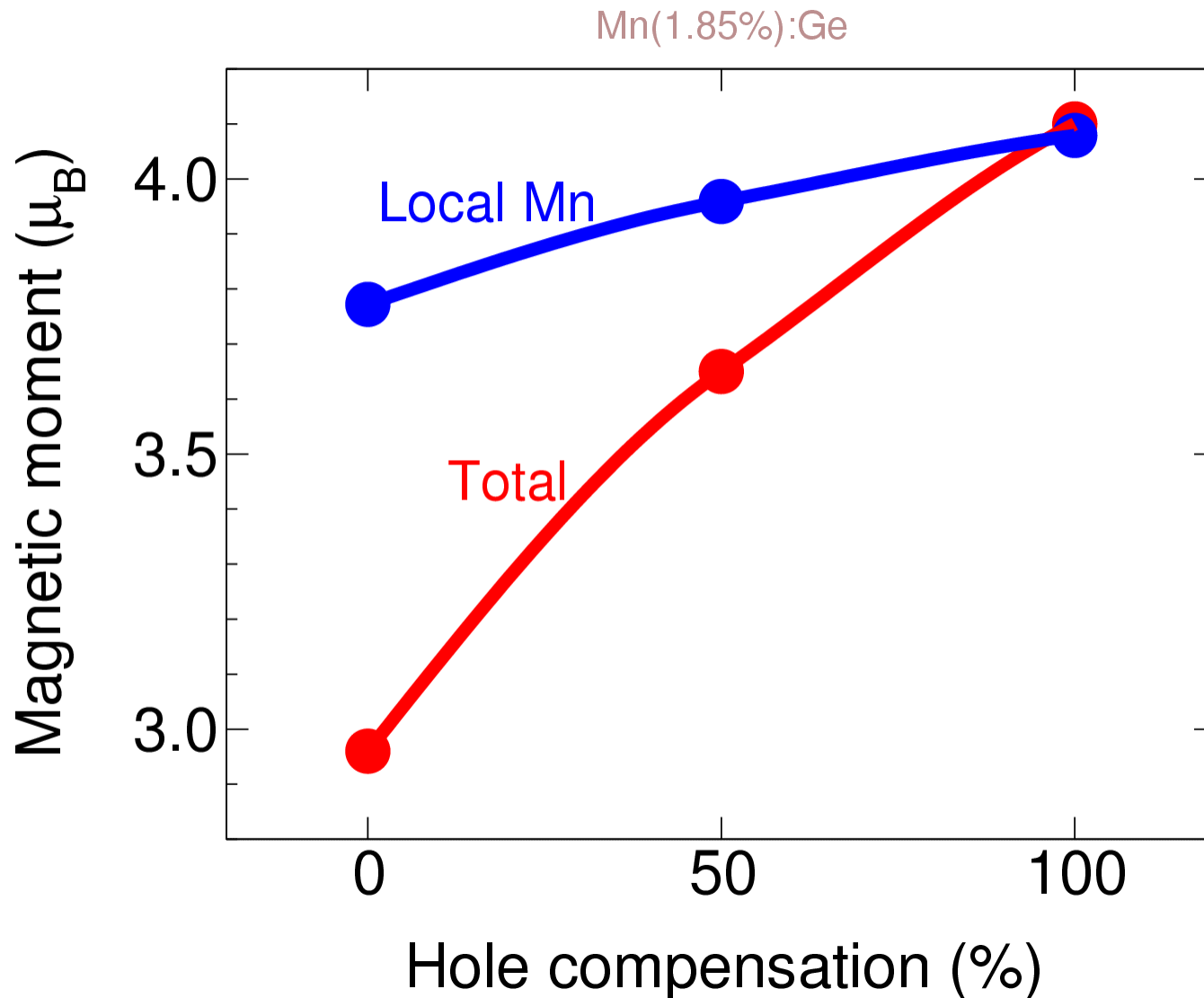
Particle-induced x-ray (PIXE) spectra

*Yu et al., Phys. Rev. B (2002)*





# The role of compensation: Mn moment





# Compensation suppresses the holes

